Analysis of Buried Arch Structures; Performance Versus Prediction

D.A. Jenkins: Reinforced Earth Pty Ltd, Somersby, NSW, Australia

Synopsis: The Reinforced Earth Group introduced the TechSpan™ arch system in 1986. Since then over 500 buried precast concrete arches have been completed to site specific designs using a design method based on finite element analysis.

In order to verify the design results the finite element analysis results have been compared with alternative analysis methods and with measurements of actual structures during construction. In this paper the results of the alternative design methods are discussed, and their predictions are compared with site measurements.

The alternative analysis procedures are found to give widely varying results, with the more refined procedures giving results closer to the finite element results. Simple analysis methods appear to greatly overestimate the effect of surcharge loading and high fills. Calculated moments and shear forces are usually much higher than those found using finite element analysis, yet they may be unconservative.

The finite element results are in excellent agreement with site measurements of deflections. Measured steel strains suggest much lower bending moments than those predicted by any of the analysis methods. The apparent discrepancy is believed to be because the concrete remained substantially uncracked during loading.

1.0 INTRODUCTION

1.1 The TechSpan™ Arch System

The TechSpan™ system consists of a two piece, three pin, buried precast concrete arch, usually constructed with Reinforced Earth™ head walls and wing walls (Figure 1). Arch dimensions are typically in the range of 5 to 20 metres span, and 3 to 8 metres height. Typical applications are road and rail crossings and tunnels and creek and small river crossings and diversions.

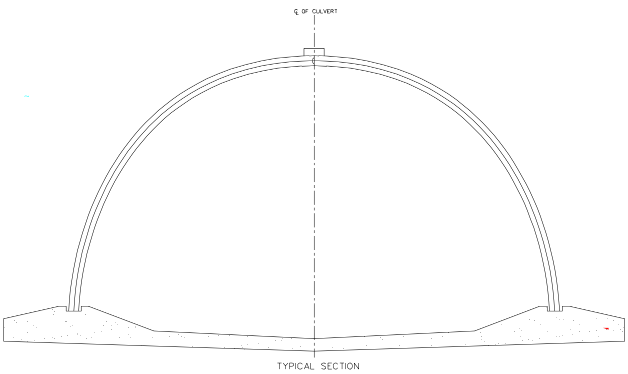

Figure 1; Typical Cross Section

Each arch is designed on a site specific basis, the arch profile being optimised to suit the needs of the client and to minimize forces in the completed structure.

1.2 The Finite Element Analysis Method

A finite element analysis is carried out on each arch. The essential features embodied in the analysis are:

- An elasto-plastic soil model with soil stiffness and Poisson's ratio related to confining pressure.
- Soil loads are applied in stages reflecting the sequence of filling employed on site.
- A layer of "friction elements" is placed between the arch and the soil, allowing the soil to slip relative to the arch.
- A compaction load is applied to each backfill layer.
- The concrete is modeled as a linear elastic material.

1.3 Alternative Analysis Methods

Alternative analysis methods considered in this paper are:

- A simple elastic analysis, applying horizontal soil loads as a fixed proportion of the vertical stress.
- A beam and spring model with compression only springs and soil loads and springs applied in stages reflecting the sequence of filling employed on site.

1.4 Site Measurements

Measurements of deflections, soil pressures, and steel stresses have been taken at four sites in Japan. This paper compares in detail the measurements taken at the Oita TechSpan Project site with predicted values.

2.0 THE FINITE ELEMENT ANALYSIS

2.1 Finite Element Model

The finite element model for a typical arch is shown in Figure 2. Eight noded plain strain plate elements are used for both soil and concrete elements. Thin friction elements are provided around the arch, and hinge elements at the base and crown allow rotation.

The model is built up in stages reflecting the actual backfill sequence; starting with the arch and foundation alone, then adding soil elements to each side of the arch alternately. For each stage a compaction load is applied then removed, so that only "locked in" compaction stresses and strains are added in to the next stage of the analysis.

2.2 Materials Properties

The soil model is based on the hyperbolic model developed by Duncan and Chang (Ref. 1). The soil stiffness (Young's Modulus, E) and Poisson's Ratio, μ, are related to the confining pressure, $\sigma_{3,}$ by the four "Duncan constants": K, n, K_b, and m:

Young's Modulus	$E/ P_a = K(\sigma_3/ P_a)^n$
Poisson's Ratio	$\mu = 0.5 - E/6B$
Where	$B/ P_a = K_b(\sigma_3/P_a)^m$

The value of these constants may be determined from triaxial testing, or typical values for compacted select fill may be assumed.

The soil parameters used in a standard TechSpan analysis for a typical select fill material are:

K = 500, Kb = 300, m = 0.2, n = 0.4

Hence for $\sigma_3 = P_a = 100$ kPa, E = 50 Mpa and $\mu = 0.22$

The soil failure stress is based on the Mohr - Coulomb criterion, using a friction angle of 30 degrees and no cohesion.

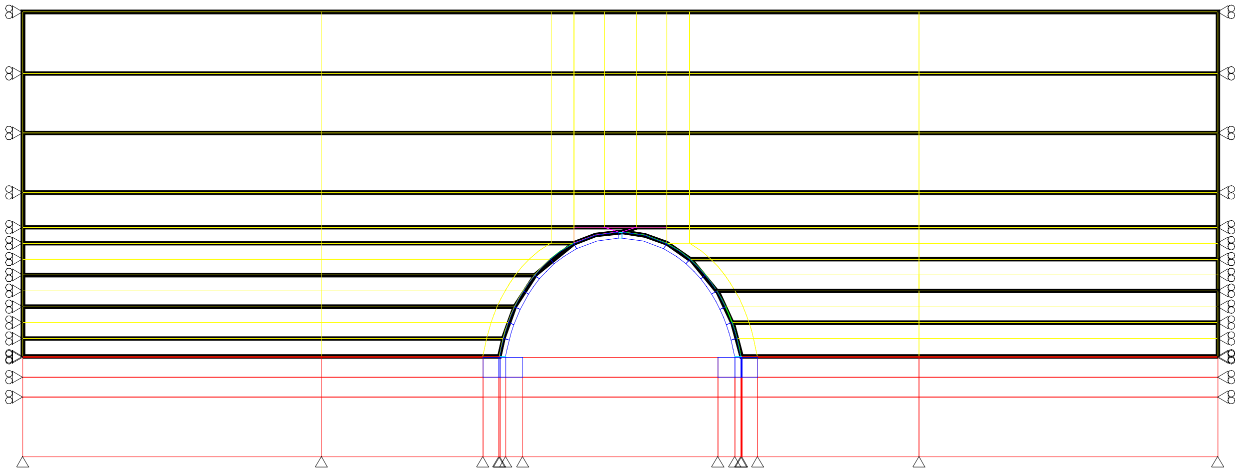

Figure 2: Finite Element Mesh

The concrete elements are assumed to be linear elastic in the standard analysis, with an E value of 20 GPa.

In this paper the finite element analysis was repeated using modified concrete E values of 35 GPa, and soil cohesion of 20 kPa. The values were selected to improve the correlation with the deflections measured on site.

3.0 THE SIMPLE ELASTIC ANALYSIS

3.1 The Arch Model

The arch is modeled as a linear elastic member pinned at the base and at the crown. Soil loads are applied as a series of horizontal and vertical point loads, dependent on the depth of fill and the specified horizontal pressure coefficient, K. No attempt is made to model the interaction of the arch and the surrounding soil or the effect of foundation settlement. Because the arch is statically determinate it is possible to carry out this analysis quickly and easily with a spreadsheet.

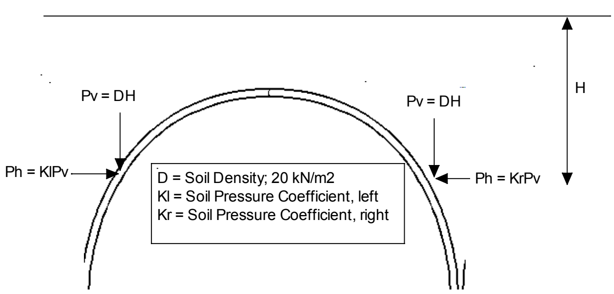

Figure 3: Simple Elastic Analysis

3.2 Soil Loads

The vertical soil stress is taken to be the weight of soil plus surcharge over the point in question. In this paper three alternative assumptions have been made for the horizontal pressure coefficient, K:

- Active pressure conditions, with a backfill friction angle of 36º, resulting in a constant K value of 0.26
- "At rest" conditions, with a backfill friction angle of 30º, resulting in a constant K value of 0.5
- Assumed maximum and minimum K values of 0.6 and 0.2 on opposite sides of the arch (Figure 3). This is an extremely conservative assumption, giving rise to a pressure distribution that could not arise in practice on a relatively flexible structure such as a three pinned arch. Nonetheless this sort of pressure distribution is commonly applied to rigid buried structures (such as box culverts) and some authorities have imposed similar requirements for the analysis of buried arches.

4.0 THE BEAM AND SPRING MODEL

4.1 The Arch and Soil Model

In the beam and spring model the interaction between the arch and the surrounding soil is modeled by means of a series of springs around the arch (Figure 4). In this model horizontal and vertical linear elastic springs are placed at each node, and are only active in compression.

4.2 Spring Properties

The spring stiffness values used in this paper are based on recommendations given by Bowles for use in the analysis of foundations on elastic soils - (Ref 2). Recommended values in this text lie between about 4000 kN/m3 and 130000 kN/m3. This range has been reduced by comparison of the results of analyis to the measured values to:

- A value of 4000 kN/m^3 based on an equation attributed to Vesic - (Ref 3), treating the arch height as being equivalent to the width of a footing, and assuming a soil E value of 30 MPa.

- A value of 20 000 kN/m^3 being the lower end of the range recommended for silty medium dense sand.

A spring stiffness value of 50 000 kN/m^3 was also considered, but was not used because the predicted deflections did not compare well with the values measured on site.

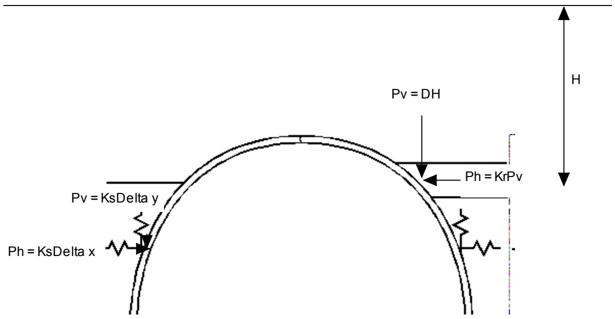

Figure 4: Beam and Spring Analysis

4.3 Soil Loads

The vertical soil stress is taken to be the weight of soil plus surcharge over the point in question, as for the simple elastic model. Since the effect of arch movements is modeled by the springs the horizontal pressure coefficient is taken to be equal to the Rankine active pressure coefficient, with a backfill friction angle of 36º, resulting in a K value of 0.26.

5.0 SITE MEASUREMENT RESULTS

5.1 The Oita TechSpan

Deflection measurements were taken at the locations shown in Figure 5 at 7 cross sections. The average deflections from all seven sections are plotted in Figures 6 and 7. The horizontal deflections, DX1 to DX3, are the change in width at the indicated levels. The vertical deflections, DY1 and DY2, are the absolute change in level at the base of the arch, and the DY3 value is the vertical crown deflection relative to the base.

Reinforcement strains were also measured at each section. The derived bending moments were much less than those found in any of the analyses. Reasons for this apparent anomaly are discussed later in this paper.

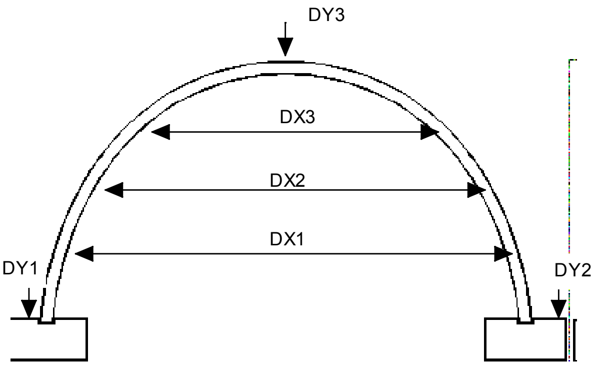

Figure 5: Location of Deflection Measurements

6.0 RESULTS OF THE ARCH ANALYSIS

For each set of analyses graphs have been plotted of:

- the vertical (DY3) and horizontal (DX4) deflection of the arch crown against fill height (e.g. Figure 8)

- the change in arch width (DX1 to DX3) against fill height (e.g. Figure 9)

- the envelope of maximum and minimum moments around the arch, over all fill heights together with the bending moment at maximum fill height (e.g. Figure 10)

The measured relative vertical deflection at the crown, and change in arch width are also plotted. Measured horizontal deflections at the crown are not available for this project; based on observations at other projects horizontal deflections are expected to be less than 5 mm during backfill and negligible under live load.

6.1 The Simple Elastic Analysis

Graphs have been plotted for each of the three assumptions for the value of the horizontal soil pressure coefficient, K. These graphs are shown in Figures 8 to 10 for K = 0.26, Figures 11 to 13 for K = 0.5, and Figures 14 to 16 for K left = 0.2, K right = 0.6.

6.2 Simple Elastic Results

The important features of the results of the arch analysis are summarised below:

For K = 0.26

- Vertical deflections of the crown during initial backfill are upwards and close to but less than the measured values (Figure 8).

- When the fill reaches the arch crown the arch starts to deflect downward at a much greater rate than the measured values, finishing with a downward deflection of 30 mm.

- Horizontal deflections at the crown during backfill are consistent with experience, but the deflection under live load surcharge (8 mm) is much greater than is found in practice.

- The change in arch width follows the pattern of the vertical deflection, being in the same direction as the measured values, but slightly smaller during initial backfill and much greater during the later stages (Figure 9).

- The calculated maximum positive moment (tension inner face) of about 100 kNm occurs when the backfill is at the arch crown. This value is the least found in any of the analyses. A very large negative moment (tension outer face) of -300 kNm is found at completion of backfill, and surcharge loading increases this moment to -400 kNm (Figure 10).

For K = 0.5

- Vertical deflections of the crown are close to the measured values throughout the backfill sequence (Figure 11).

- Horizontal deflections at the crown during backfill are slightly higher than expected and the deflection under live load surcharge (12 mm) is much greater than found in practice.

- The change in arch width follows the pattern of the vertical deflection, being close to the measured values throughout the backfill sequence (Figure 12).

- The calculated maximum moment (200 kNm) occurs when the backfill is at the arch crown, and is almost double the magnitude found for K = 0.26. The moment at completion of backfill is very low (maximum 15 kNm), but surcharge loading generates a negative moment of over -100 kNm (Figure 13).

For K Left = 0.2, K Right = 0.6

- Vertical deflections of the crown are close to the measured values throughout the backfill sequence (Figure 14).

- Horizontal deflections at the crown during backfill and under live load surcharge are very much greater than found in practice. The final horizontal deflection of 90 mm to the left is clearly incompatible with observations of completed structures, and with the development of the assumed horizontal loads.

- The change in arch width is in the same direction as the measured values, but slightly smaller during initial backfill and much greater during the later stages (Figure 15).

- A very large maximum moment (700 kNm) occurs in one half of the arch when the backfill is at the arch crown. A negative moment of similar magnitude is found at completion of backfill in the other segment. Surcharge loading increases the magnitude of the moment in both arch segments by over 100 kNm (Figure 16).

6.3 The Beam and Spring Model

Graphs have been plotted for each of the two assumptions for the value of the spring stiffness coefficient, K_s. These graphs are shown in Figures 17 to 19 for K_s = 4 000 kN/m^3, and Figures 20 to 22 for K_s = 20 000 kN/m^3.

6.4 Beam and Spring Results

For K_s = 4 000 kN/m^3

- Vertical deflections of the crown are very close to the measured values at the start and end of the backfill sequence, but are about 50% greater when the backfill is at the arch crown (Figure 17).

- Horizontal deflections at the crown during backfill and the deflection under live load surcharge (2 mm) are of the expected magnitude.

- The change in arch width follows the pattern of the vertical deflection, being close to the measured values at the start and end of the backfill sequence, but about 50% greater during the middle phase (Figure 18).

- The calculated maximum moment (130 kNm) occurs when the backfill is at the arch crown. The moment at completion of backfill is a low negative moment (-60 kNm), and surcharge loading generates a comparatively small moment (Figure 19).

For K_s = 20 000 kN/m^3

- Vertical deflections of the crown are very close to the measured values at the start of the backfill sequence, but are about 75% greater when the backfill is at the arch crown, and downward deflection in the later stages of backfill are only about 50 % of the measured values (Figure 20).

- Horizontal deflections at the crown during backfill and the deflection under live load surcharge (1 mm) are of the expected magnitude, and smaller than for K_s = 4000 kN/m^3.

- The change in arch width follows the pattern of the vertical deflection, being close to the measured values at the start of the backfill sequence, but about 50% greater during the middle phase, with a smaller relative change in the final stages (Figure 21).

- The calculated maximum moment (150 kNm) occurs when the backfill is at the arch crown. The moment at completion of backfill remains positive, and is about 33% less than the maximum value. Surcharge loading generates only a small moment (Figure 22).

6.5 The Finite Element Model

Graphs have been plotted for each of the two assumptions for the soil parameters. These graphs are shown in Figures 23 to 25 for standard properties, and Figures 26 to 28 for modified properties.

6.6 Finite Element Results

For standard properties:

- Vertical deflections of the crown follow a similar path to the measured values, but are up to 75% greater when the backfill is at the arch crown (Figure 23).

- Horizontal deflections at the crown during backfill and the deflection under live load surcharge (1 mm) are of the expected magnitude.

- The change in arch width follows the pattern of the vertical deflection, being about 70% greater than the measured deflections during the middle phase. The final deflections are very close to the measured value (Figure 24).

- The calculated maximum moment (100 kNm) occurs when the backfill is at the arch crown. The maximum moment at completion of backfill remains positive, and is about 35% less than the maximum value. Due to the use of non-linear soil properties and asymmetric loading the final moment around the arch is not symmetrical, and a negative moment of -50 kNm has developed on the left hand side. Surcharge loading generates only a small moment (Figure 25).

For modified properties:

- Vertical deflections of the crown follow the measured values very closely for fill heights up to the arch crown. For increasing fill heights the slope of the calculated and measured deflection curves are very similar but offset by about 2 mm (Figure 26).

- Horizontal deflections at the crown during backfill and the deflection under live load surcharge (1 mm) are of the expected magnitude.

- The change in arch width follows the pattern of the vertical deflection, being close to the measured values during filling up to the arch crown, and having a similar slope to the measured values in the final phase of filling (Figure 27).

- The calculated maximum moment (120 kNm) occurs when the backfill is at the arch crown, and is about 35% lower than the moment found using standard soil properties. The maximum moment at completion of backfill remains positive, and is almost 60% less than the maximum value. As for the standard soil properties the final moment around the arch is not symmetrical, and a negative moment of about 60 kNm has developed on the left hand side. Surcharge loading generates only a small moment (Figure 28).

7.0 CONCLUSIONS

7.1 Simple Elastic Model

The deflections predicted by the linear elastic model vary over a wide range, with final crown deflections varying between +3 mm and -30 mm vertically and between 0 mm and -90 mm horizontally. The results derived using a K value of 0.5 are at first sight remarkably close to the measured values, however the close fit is related to the way in which this particular arch shape was defined, and cannot be taken as evidence of accuracy of the analysis method for general application. This point is discussed further below.

Although measurements of the absolute horizontal crown deflection were not available it is clear that the calculated horizontal displacement under surcharge loading of about 10 mm is much greater than occurs in practice. Where unequal K values have been used a horizontal displacement of up to 90 mm is found. It is clearly impossible for the assumed pressure distribution to develop since such a large displacement will develop active and passive pressures in the opposite direction to the assumed loading.

The range of calculated moments is even more extreme than the deflections, with maximum moments ranging between 100 kNm and 850 kNm. It cannot even be assumed that the results are conservative, since the calculated maximum moment at completion of backfill varies between +10 kNm and -750 kNm.

The reason for the very low calculated moment when K = 0.5 is related to the arch shape. TechSpan arches are designed using funicular curve theory so that under one specified loading condition the moments around the arch due to soil loads will be zero. The Oita arch was designed for a fill height of 17.5 metres and a K value of 0.5. It is therefore no accident that when this particular load is applied to the arch the resulting moments are indeed close to zero. For the same reason the final deflection of the arch will also be close to zero, both in theory and (if the actual final pressure distribution is reasonably close to that assumed in the design) in practice.

In summary the simple elastic method cannot be considered a suitable basis for arch design. Calculated moments assuming a single K value may be unsafe, but the results of using unequal K values are grossly over conservative.

7.2 The Beam and Spring Model

The results of the beam and spring model analysis are markedly superior to the simple analysis. Using the lower bound spring stiffness the calculated deflections are in excellent agreement with the measured values, and the calculated moments are in reasonable agreement with the finite element results.
However change in the assumed spring stiffness has a marked effect on the calculated deflections and moments, particularly for the final moments. Since the range of stiffness values used in this paper (4 000 kN/m^3 to 20 000 kN/m^3) is much smaller than the range of commonly used values (up to 130 000 kN/m^3) the selection of a suitable spring stiffness may be difficult to justify. Trial analyses using a stiffness of 50 000 kN/m^3 (which is within the range commonly used in similar applications) for instance gave very poor results compared with the measured deflections. Since the appropriate spring stiffness is related to the height of the structure the results reported in this paper may not be reliably extrapolated to other structures.

7.3 The Finite Element Analysis

The standard properties for the finite element analysis use a concrete stiffness of 20 GPa for reasons discussed below. This value is lower than used in the other analyses, which used a more typical value for a Grade 40 concrete of 35 GPa. For this reason the standard finite element analysis resulted in higher deflections than the measured values, particularly when the arch was mostly unrestrained by the surrounding soil. When the concrete stiffness was increased to 35 GPa, and the soil cohesion increased from 0 to 20 Kpa, the calculated deflections were in excellent agreement with the measured values.

The bending moments found in the two finite element runs are in good agreement, with the modified soil properties and higher concrete E value giving slightly higher moments.

7.4 Measured Moments

The maximum measured moments with fill to the top of the arch was 30 kNm, which is much lower than any of the analysis results, even though several of the analyses gave very close agreement with the measured deflections. The discrepancy is probably due to the concrete remaining substantially uncracked under loading, so that most of the tensile bending stress is carried by the concrete. Based on the assumption that the concrete was uncracked the total maximum moment consistent with the measured steel strain would be approximately 110 kNm, which is in good agreement with the finite element results.

7.5 Standard Design Concrete E values

The measured deflections would appear to indicate that the concrete E value of 20 GPa used in the standard design procedure should be increased to about 35 GPa. However the purpose of the analysis, in the design context, is to check that the loads in the arch under ultimate conditions do not exceed the capacity of the section. It is clearly unrealistic to assume that the concrete will remain uncracked at the ultimate capacity of the section, except under very high axial loads. The assumption of an E value of 20 GPa can therefore be seen as conservative, since the effective E value of a typically reinforced cracked concrete section would be in the range of 5 - 10 GPa.

8.0 ACKNOWLEDGEMENTS

The author wishes to thank the Kawasho Corporation for their permission to publish the results of their site monitoring and measurements.

9.0 REFERENCES

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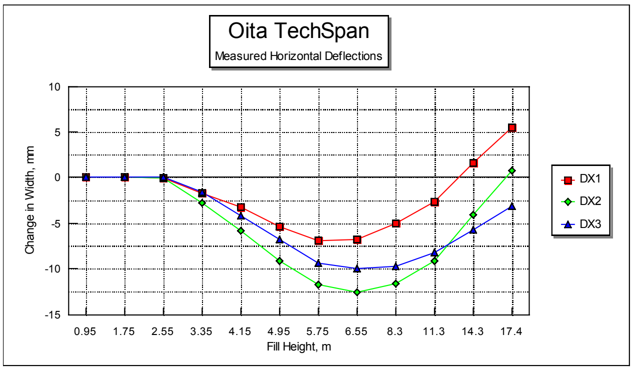

Figure 6: Measured Horizontal Deflections

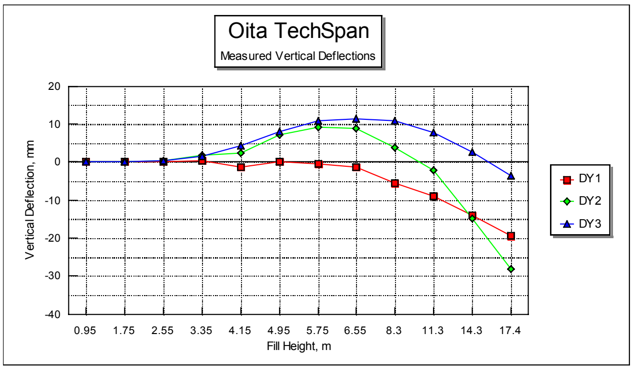

Figure 7; Measured Vertical Deflections

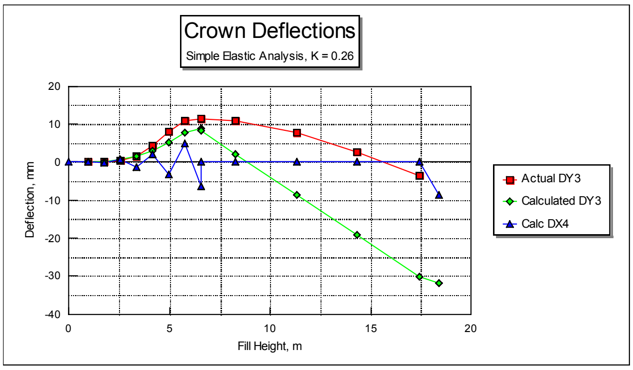

Figure 8: Crown Deflections; K = 0.26

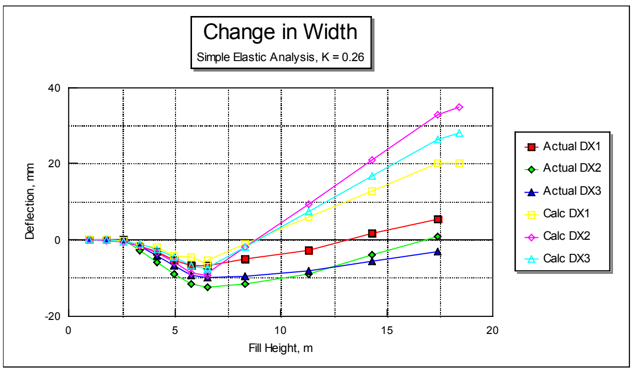

Figure 9: Change in Width, K = 0.26

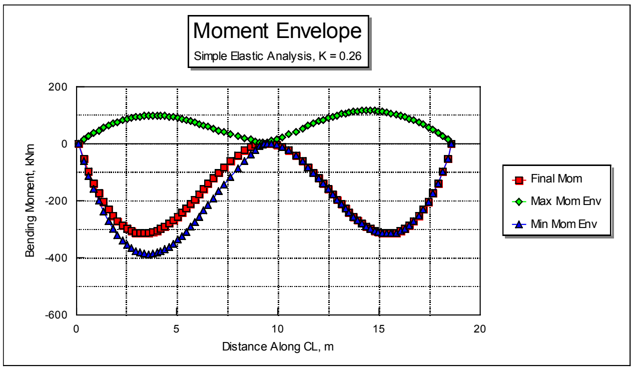

Figure 10: Moment Envelope; K = 0.26

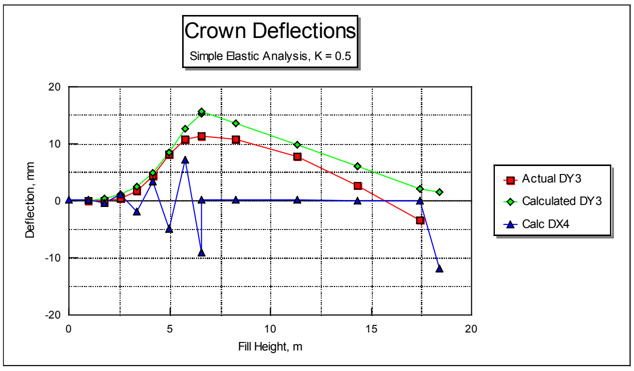

Figure 11; Crown Deflections, K = 0.5

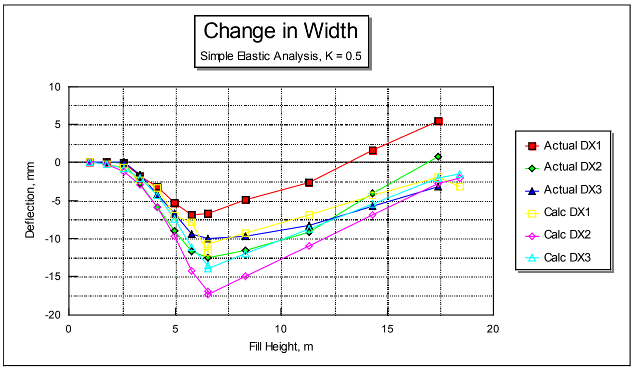

Figure 12: Change in Width; K = 0.5

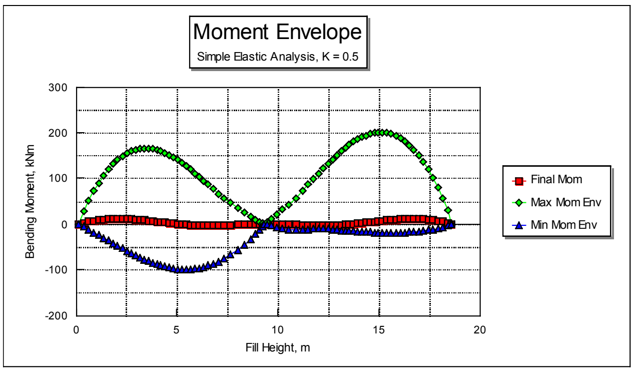

Figure 13; Moment Envelope, K = 0.5

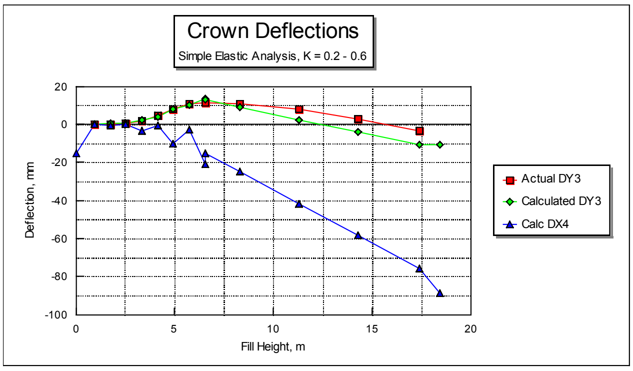

Figure 14: Crown Deflection, K = 0.2 & 0.6

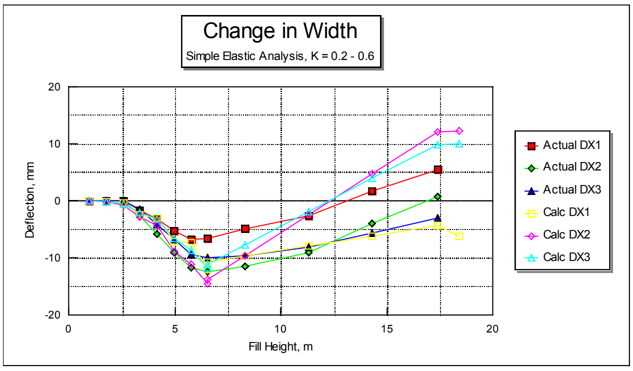

Figure 15: Change in Width, K = 0.2 & 0.6

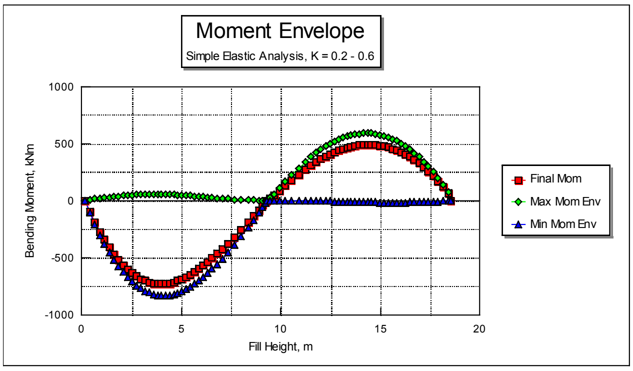

Figure 16: Moment Envelope: K = 0.2 & 0.6

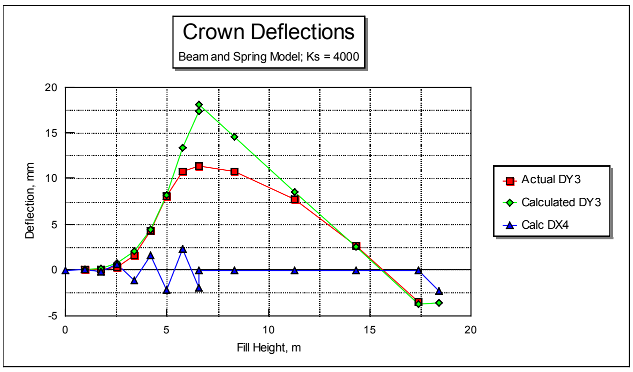

Figure 17: Crown Deflection, Ks = 4000

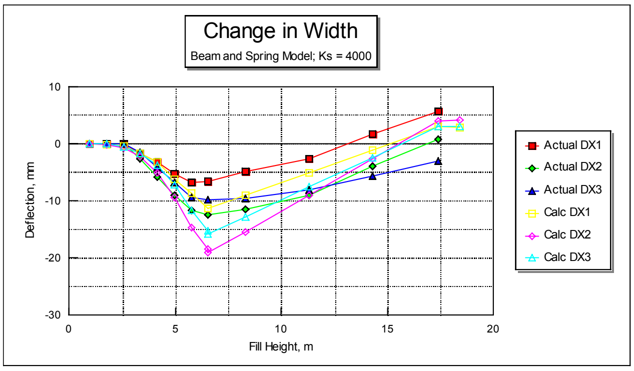

Figure 18; Change in Width, Ks = 4000

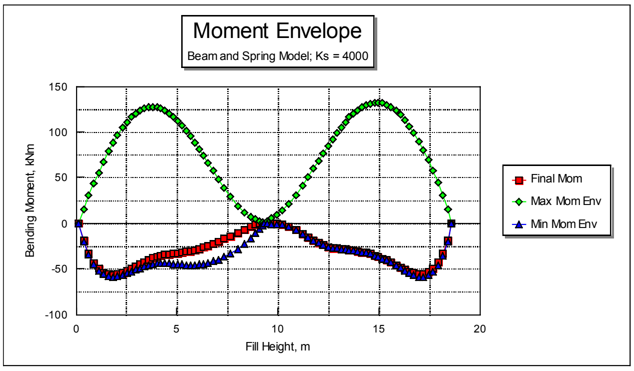

Figure 19; Moment Envelope; Ks = 4000

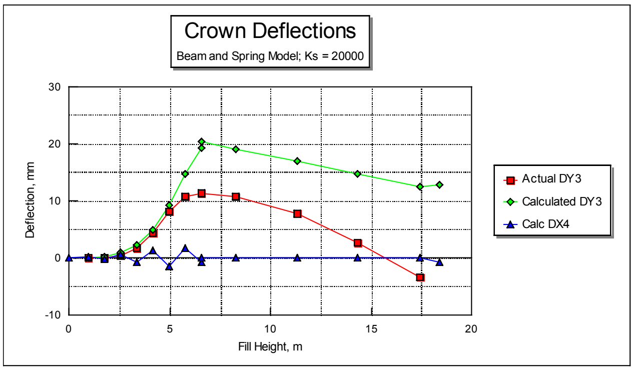

Figure 20; Crown Deflection, Ks = 20000

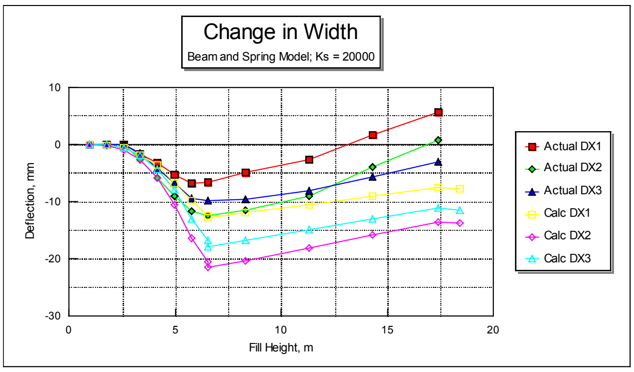

Figure 21: Change in Width, Ks = 20000

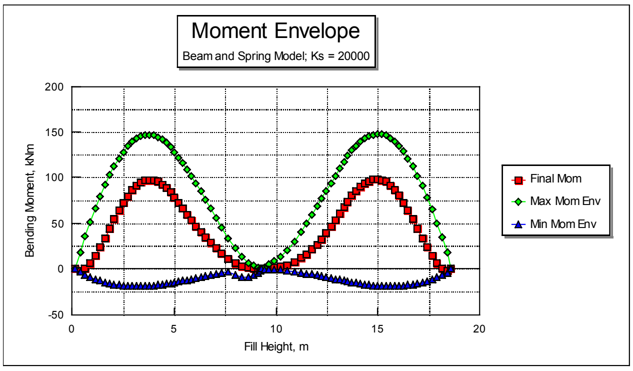

Figure 22: Moment Envelope, Ks = 20000

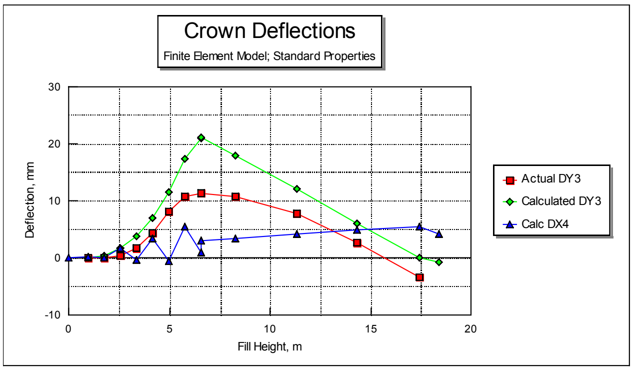

Figure 23: Crown Deflections, FEA 1

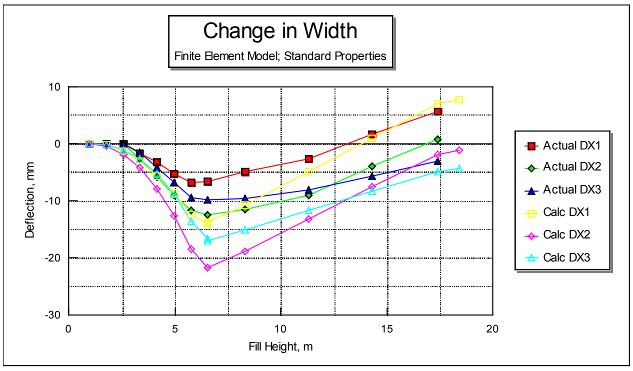

Figure 24: Change in Width, FEA 1

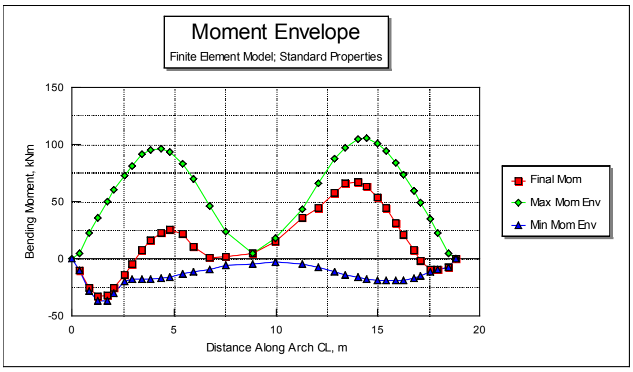

Figure 25: Moment Envelope, FEA 1

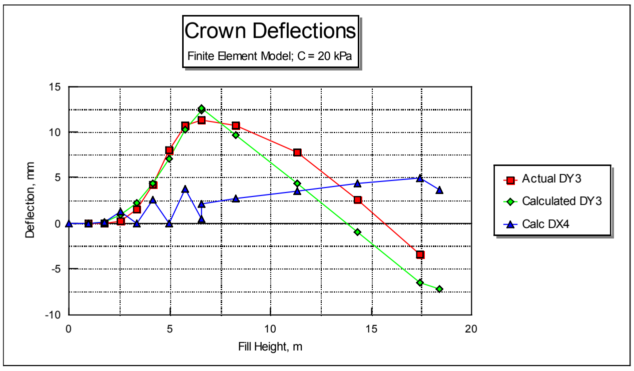

Figure 26: Crown Deflections, FEA 2

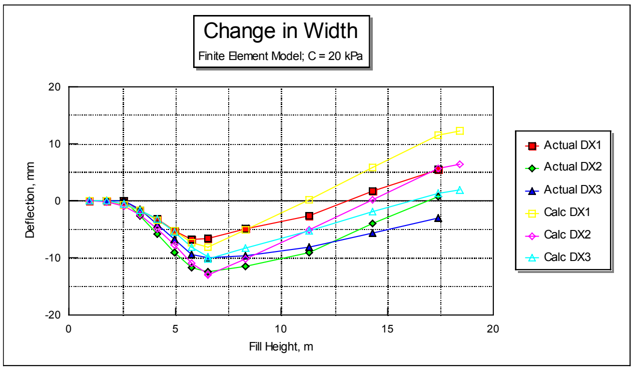

Figure 27: Change in Width, FEA 2

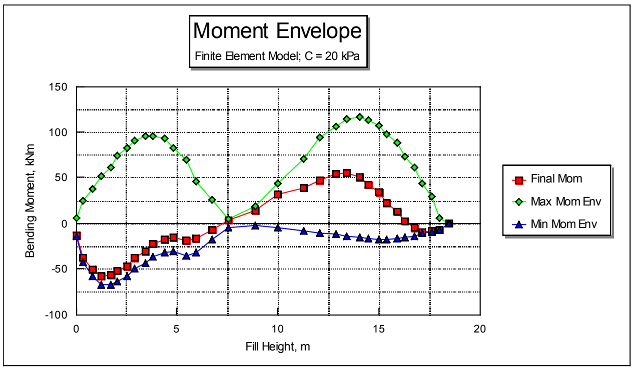

Figure 28: Moment Envelope, FEA2